ON INTRA-REGULAR LEFT ALMOST SEMIHYPERGROUPS WITH PURE LEFT IDENTITY

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Abstract. In this paper, we characterize intra-regular LA-semihypergroups by using the properties of their left and right hyperideals and we investigate some useful conditions for an LA-semihypergroup to become an intra-regular LA-semihypergroup.

1. Introduction

Kazim and Naseerudin [17], introduced the concept of left almost semigroups (abbreviated as LA-semigroups), right almost semigroups (abbreviated as RA-semigroups). They generalized some useful results of semigroup theory. Later, Mushtaq [21, 22] and others further investigated the structure and added many useful results to the theory of LA-semigroups, also see [15, 16, 18, 25, 26]. An LA-semigroup is the midway structure between a commutative semigroup and a groupoid. Despite the fact, the structure is non-associative and non-commutative. It nevertheless possesses many interesting properties which we usually find in commutative and associative algebraic structures. Mushtaq and Yusuf produced useful results [23], on locally associative LA-semigroups in 1979. In this structure they defined powers of an element and congruences using these powers. They constructed quotient LA-semigroups using these congruences. It is a useful non-associative structure with wide applications in theory of flocks. A several papers are written on LA-semigroups. There are lot of results which have been added to the theory of LA-semigroups by Mushtaq, Kamran, Shabir, Aslam, Davvaz, Madad, Faisal, Abdullah, Yaqoob, Hila, Rehman, Chinram, Holgate, Jezek, Protic and many other researchers.

Hyperstructure theory was introduced in 1934, when F. Marty [20] defined hypergroups, began to analyze their properties and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science and they are studied in many countries of the world. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. A lot of papers and several books have been written on hyperstructure theory, see [4, 27]. A recent book on hyperstructures [3] points out on their applications in rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs and hypergraphs. Another book [7] is devoted especially to the study of hyperring

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theory. Several kinds of hyperrings are introduced and analyzed. The volume ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures: e-hyperstructures and transposition hypergroups. Many authors studied different aspects of semihypergroups, for instance, Corsini et al. [1, 2], Davvaz et al. [5, 6], Drbohlav et al. [8], Fasino and Freni [9], Gutan [10], Hasankhani [11], Hedayati [12], Hila et al. [14], Leoreanu [19] and Onipchuk [24]. Recently, Hila and Dine [13] introduced the notion of LA-semihypergroups as a generalization of semigroups, semihypergroups and LA-semigroups. They investigated several properties of hyperideals of LA-semihypergroup and defined the topological space and study the topological structure of LA-semihypergroups using hyperideal theory.

In this paper, we shall prove some results on intra-regular LA-semihypergroups.

2. **Preliminaries and Basic Definitions**

In this section, we recall certain definitions and results needed for our purpose.

Definition 1. *A map $\circ : H\times H \to \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set H, where H is a non-empty set and $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\{\emptyset\}$ denotes the set of all non-empty subsets of H. A hypergroupoid is a set H together with a (binary) hyperoperation.*

If A and B be two non-empty subsets of H, then we denote

$$A \circ B = \bigcup_{a\in A, b\in B} a\circ b, \quad a\circ A = \{a\}\circ A \text{ and } a\circ B = \{a\}\circ B.$$

Definition 2. [13] *A hypergroupoid $(H,\circ)$ is called an LA-semihypergroup if, for all $x,y,z \in H$,*

$$(x\circ y)\circ z = (z\circ y)\circ x.$$

The law $(x\circ y)\circ z = (z\circ y)\circ x$ is called a left invertive law.

Example 1. *Let $H = \{x, y, z, w, t\}$ with the binary hyperoperation defined below:*

$\circ$	x	y	z	w	t
x	x	x	x	x	x
y	x	$\{z,t\}$	z	$\{x,w\}$	$\{z,t\}$
z	x	z	z	$\{x,w\}$	z
w	x	$\{x,w\}$	$\{x,w\}$	w	$\{x,w\}$
t	x	$\{y,t\}$	z	$\{x,w\}$	$\{y,t\}$

Clearly H is not a semihypergroup because $\{y,z,t\} = (t\circ t)\circ y \neq t\circ(t\circ y) = \{y,t\}$. Thus H is an LA-semihypergroup because the elements of H satisfies the left invertive law.

Example 2. *Let $H = \mathbb{Z}$. If we define $x\circ y = y - x + 3\mathbb{Z}$, where $x, y \in \mathbb{Z}$. Then $(H,\circ)$ becomes an LA-semihypergroup as,*

$$\begin{aligned}(x\circ y)\circ z &= z-(x\circ y)+3\mathbb{Z} = z-(y-x+3\mathbb{Z})+3\mathbb{Z}\\ &= z-y+x-3\mathbb{Z}+3\mathbb{Z} = x-y+z-3\mathbb{Z}+3\mathbb{Z}\\ &= x-(y-z+3\mathbb{Z})+3\mathbb{Z} = x-(z\circ y)+3\mathbb{Z}\\ &= (z\circ y)\circ x.\end{aligned}$$

Which implies that $(x \circ y) \circ z = (z \circ y) \circ x$ holds for all $x, y, z \in \mathbb{Z}$, also it is clear that $(x \circ y) \circ z \neq x \circ (y \circ z)$. Hence $(H, \circ)$ is an LA-semihypergroup.

Every LA-semihypergroup satisfies the law

$$(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w)$$

for all $x, y, z, w \in H$. This law is known as medial law. (cf. [13]).

Definition 3. *Let H be an LA-semihypergroup. An element $e \in H$ is called*

(i) left identity (resp. pure left identity) if for all $a \in H$, $a \in e \circ a$ (resp. $a = e \circ a$),

(ii) right identity (resp. pure right identity) if for all $a \in H$, $a \in a \circ e$ (resp. $a = a \circ e$),

(iii) identity (resp. pure identity) if for all $a \in H$, $a \in e \circ a \cap a \circ e$ (resp. $a = e \circ a \cap a \circ e$).

Example 3. *Let $H = \{x, y, z, w\}$ with the binary hyperoperation defined below:*

$\circ$	x	y	z	w
x	x	y	z	w
y	z	$\{y, z\}$	$\{y, z\}$	w
z	y	$\{y, z\}$	$\{y, z\}$	w
w	w	w	w	H

Clearly H is an LA-semihypergroup because the elements of H satisfies the left invertive law. Here x is a pure left identity because for all $a \in H$, $a = x \circ a$. And in Example 1, one can see that t is a left identity but not a pure left identity.

Lemma 1. *Let H be an LA-semihypergroup with pure left identity e, then $x \circ (y \circ z) = y \circ (x \circ z)$ holds for all $x, y, z \in H$.*

Proof. Let H be an LA-semihypergroup with pure left identity e, then for all $x, y, z \in H$ and by medial law, we have

$$x \circ (y \circ z) = (e \circ x) \circ (y \circ z) = (e \circ y) \circ (x \circ z) = y \circ (x \circ z).$$

This completes the proof. □

Lemma 2. *Let H be an LA-semihypergroup with pure left identity e, then $(x \circ y) \circ (z \circ w) = (w \circ y) \circ (z \circ x)$ holds for all $x, y, z, w \in H$.*

Proof. Let H be an LA-semihypergroup with pure left identity e, then for all $x, y, z \in H$ and by medial law, we have

$$\begin{aligned}(x \circ y) \circ (z \circ w) &= ((e \circ x) \circ y) \circ ((e \circ z) \circ w) = ((y \circ x) \circ e) \circ ((w \circ z) \circ e) \\ &= ((y \circ x) \circ (w \circ z)) \circ (e \circ e) = ((e \circ e) \circ (w \circ z)) \circ (y \circ x) \\ &= (e \circ (w \circ z)) \circ (y \circ x) = (w \circ z) \circ (y \circ x) \\ &= (w \circ y) \circ (z \circ x).\end{aligned}$$

This completes the proof. □

The law $(x \circ y) \circ (z \circ w) = (w \circ y) \circ (z \circ x)$ is called a paramedial law.

An element 0 in an LA-semihypergroup H is called zero element if $x \circ 0 = 0 \circ x = \{0\}$, for all $x \in H$. Let H be an LA-semihypergroup. A non-empty subset A of H is called a sub LA-semihypergroup of H if $x \circ y \subseteq A$ for every $x, y \in A$. A subset I of an LA-semihypergroup H is called a right (left) hyperideal if $I \circ H \subseteq I$ $(H \circ I \subseteq I)$ and is called a hyperideal if it is two-sided hyperideal, and if I is a left

hyperideal of H, then $I \circ I = I^2$ becomes a hyperideal of H. By a bi-hyperideal of an LA-semihypergroup H, we mean a sub LA-semihypergroup B of H such that $(B \circ H) \circ B \subseteq B$. A sub LA-semihypergroup B of H is called a (1,2)-hyperideal of H if $(B \circ H) \circ B^2 \subseteq B$. It is easy to note that each right hyperideal is a bi-hyperideal. If $E(B_H)$ denotes the set of all idempotents subsets of H with pure left identity e, then $E(B_H)$ forms a hypersemilattice structure. The intersection of any set of bi-hyperideals of an LA-semihypergroup H is either empty or a bi-hyperideal of H. A sub LA-semihypergroup T of H is called an interior hyperideal of H if $(T \circ H) \circ T \subseteq T$. A non-empty subset Q of an LA-semihypergroup H is called a quasi hyperideal of H if $Q \circ H \cap H \circ Q \subseteq Q$.

Lemma 3. *If H is an LA-semihypergroup with left identity e, then $H \circ H = H$.*

Proof. If H is an LA-semihypergroup with left identity e. Then $x \in H$ implies that

$$x \in e \circ x \subseteq H \circ H \text{ and so } H \subseteq H \circ H.$$

That is $H = H \circ H$. □

Corollary 1. *If H is an LA-semihypergroup with pure left identity e then $H \circ H = H$ and $H = e \circ H = H \circ e$.*

Proof. The proof is similar to the proof of Lemma 3. □

In an LA-semigroup every right identity becomes a left identity. But in an LA-semihypergroup every right identity need not to be a left identity. For this, let $H = \{x, y, z\}$ with the binary hyperoperation defined below:

$\circ$	x	y	z
x	$\{x, z\}$	z	$\{y, z\}$
y	$\{y, z\}$	z	z
z	$\{y, z\}$	$\{y, z\}$	$\{y, z\}$

Clearly H is not a semihypergroup because $\{y, z\} = (y \circ y) \circ z \neq y \circ (y \circ z) = \{z\}$. Thus H is an LA-semihypergroup because the elements of H satisfies the left invertive law. Here x is a right identity but not a left identity. However if an LA-semihypergroup H has a pure right identity e then, e becomes a pure left identity. For this, consider $b \in H$ and e be a pure right identity of H, then

$$b = b \circ e = (b \circ e) \circ e = (e \circ e) \circ b = e \circ b.$$

This shows that in an LA-semihypergroup every pure right identity becomes a pure left identity. Every LA-semihypergroup with pure right identity becomes a commutative hypermonoid.

Theorem 1. *An LA-semihypergroup H is a semihypergroup if and only if $a \circ (b \circ c) = (c \circ b) \circ a$ holds for all $a, b, c \in H$.*

Proof. Let H be a semihypergroup. Then we have $(a \circ b) \circ c = a \circ (b \circ c)$ for all $a, b, c \in H$, but $(a \circ b) \circ c = (c \circ b) \circ a$, thus

$$a \circ (b \circ c) = (c \circ b) \circ a \quad \text{for all } a, b, c \in H.$$

On the other hand, suppose $a \circ (b \circ c) = (c \circ b) \circ a$ holds for all $a, b, c \in H$. Since H is an LA-semihypergroup, therefore

$$a \circ (b \circ c) = (c \circ b) \circ a = (a \circ b) \circ c.$$

Thus H is a semihypergroup. This completes the proof. □

3. Intra-Regular LA-semihypergroups

In this section, we characterized intra-regular LA-semihypergroup by using the properties of left and right hyperideals.

Definition 4. *An element a of an LA-semihypergroup H is called an intra-regular if there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$ and H is called intra-regular, if every element of H is intra-regular.*

Example 4. *Let $H = \{x, y, z, w\}$ with the binary hyperoperation defined below:*

$\circ$	x	y	z	w
x	x	$\{x, w\}$	$\{x, w\}$	w
y	$\{x, w\}$	$\{y, z\}$	$\{y, z\}$	w
z	$\{x, w\}$	y	y	w
w	w	w	w	w

Clearly H is an LA-semihypergroup because the elements of H satisfies the left invertive law. Here H is intra-regular because, $x \in (y \circ x^2) \circ z$, $y \in (z \circ y^2) \circ z$, $z \in (y \circ z^2) \circ y$, $w \in (x \circ w^2) \circ z$.

Example 5. *Let $H = \mathbb{Z}$. Define a hyperoperation $\circ$ on H by:*

$$x \circ y = \{x, y\} \cup 2\mathbb{Z} \text{ for all } x, y \in H.$$

Then for all $x, y, z \in H$, we have

$$(x \circ y) \circ z = \{x, y, z\} \cup 2\mathbb{Z} = (z \circ y) \circ x.$$

This implies that $(H, \circ)$ is a an LA-semihypergroup. Since $x \in (x \circ x) \circ x = \{x\} \cup 2\mathbb{Z}$ also $x \in (x \circ (x \circ x)) \circ x = \{x\} \cup 2\mathbb{Z}$. Thus $(H, \circ)$ is a regular as well as intra-regular LA-semihypergroup.

Definition 5. *An element a of an LA-semihypergroup H with left identity e is called a left (right) invertible if there exits $x \in H$ such that $e \in x \circ a$ ($e \in a \circ x$) and a is called invertible if it is both a left and a right invertible. An LA-semihypergroup H is called a left (right) invertible if every element of H is a left (right) invertible and H is called invertible if it is both a left and a right invertible.*

Theorem 2. *Every LA-semihypergroup H with pure left identity e is intra-regular if H is left (right) invertible.*

Proof. Let H be a left invertible LA-semihypergroup with pure left identity e, then for $a \in H$ there exists $a^{'} \in H$ such that $e \in a^{'} \circ a$. Now by using left invertive law, medial law, Lemma 1 and Lemma 3, we have

$$\begin{aligned} a &= e \circ a = e \circ (e \circ a) = (a^{'} \circ a) \circ (e \circ a) \subseteq (H \circ a) \circ (H \circ a) \\ &= (H \circ a) \circ ((H \circ H) \circ a) = (H \circ a) \circ ((a \circ H) \circ H) \\ &= (a \circ H) \circ ((H \circ a) \circ H) = (a \circ (H \circ a)) \circ (H \circ H) \\ &= (a \circ (H \circ a)) \circ H = (H \circ (a \circ a)) \circ H = (H \circ a^2) \circ H. \end{aligned}$$

Which shows that H is intra-regular. The case for right invertible can be seen in a similar way. $\square$

Theorem 3. *An LA-semihypergroup H with left identity e is intra-regular if $H \circ a = H$ or $a \circ H = H$ for all $a \in H$.*

Proof. Let H be an LA-semihypergroup such that $H \circ a = H$ holds for all $a \in H$, then $H = H^2$. Let $a \in H$, therefore by using medial law, we have

$$a \in H = (H \circ H) \circ H = ((H \circ a) \circ (H \circ a)) \circ H = ((H \circ H) \circ (a \circ a)) \circ H = (H \circ a^2) \circ H.$$

Which shows that H is intra-regular.

Let $a \in H$ and assume that $a \circ H = H$ holds for all $a \in H$, then by using left invertive law, we have

$$a \in H = H \circ H = (a \circ H) \circ H = (H \circ H) \circ a = H \circ a.$$

Thus $H \circ a = H$ holds for all $a \in H$, therefore it follows from above that H is intra-regular. □

Corollary 2. *An LA-semihypergroup H with pure left identity e is intra-regular if $H \circ a = H$ or $a \circ H = H$ for all $a \in H$.*

Corollary 3. *If H is an LA-semihypergroup such that $a \circ H = H$ holds for all $a \in H$, then $H \circ a = H$ holds for all $a \in H$.*

Theorem 4. *If H is an intra-regular LA-semihypergroup with pure left identity e, then $(B \circ H) \circ B = B \cap H$, where B is a bi-(generalized bi-) hyperideal of H.*

Proof. Let H be an intra-regular LA-semihypergroup with pure left identity e, then clearly $(B \circ H) \circ B \subseteq B \cap H$. Now let $b \in B \cap H$, which implies that $b \in B$ and $b \in H$. Since H is intra-regular so there exist $x, y \in H$ such that $b \in (x \circ b^2) \circ y$. Now by using Lemma 1, left invertive law, paramedial law and medial law, we have

$$\begin{aligned}
b &\in (x \circ b^2) \circ y = (x \circ (b \circ b)) \circ y = (b \circ (x \circ b)) \circ y = (y \circ (x \circ b)) \circ b \\
&\subseteq (y \circ (x \circ ((x \circ b^2) \circ y))) \circ b = (y \circ ((x \circ b^2) \circ (x \circ y))) \circ b \\
&= ((x \circ b^2) \circ (y \circ (x \circ y))) \circ b = (((x \circ y) \circ y) \circ (b^2 \circ x)) \circ b \\
&= ((b \circ b) \circ (((x \circ y) \circ y) \circ x)) \circ b = ((b \circ b) \circ ((x \circ y)(x \circ y))) \circ b \\
&= ((b \circ b) \circ (x^2 \circ y^2)) \circ b = ((y^2 \circ x^2) \circ (b \circ b)) \circ b \\
&= (b \circ ((y^2 \circ x^2) \circ b)) \circ b \subseteq (B \circ H) \circ B.
\end{aligned}$$

This shows that $(B \circ H) \circ B = B \cap H$. □

The converse is not true in general. For this, let us consider an LA-semihypergroup $H = \{x, y, z, w\}$ with the binary hyperoperation defined below:

$\circ$	x	y	z	w
x	y	y	$\{z, w\}$	w
y	y	y	$\{z, w\}$	w
z	$\{z, w\}$	$\{z, w\}$	z	w
w	w	w	w	w

Clearly H is an LA-semihypergroup because the elements of H satisfies the left invertive law. It is easy to see that $\{z, w\}$ is a bi-(generalized bi-) hyperideal of H such that $(B \circ H) \circ B = B \cap H$ but H has no pure left identity and also H is not an intra-regular because $x \in H$ is not an intra-regular.

Corollary 4. *If H is an intra-regular LA-semihypergroup with pure left identity, then $(B \circ H) \circ B = B$, where B is a bi-(generalized bi-) hyperideal of H.*

Theorem 5. *If H is an intra-regular LA-semihypergroup with pure left identity e, then $(H \circ B) \circ H = H \cap B$, where B is an interior hyperideal of H.*

Proof. Let H be an intra-regular LA-semihypergroup with left identity e, then clearly $(H \circ B) \circ H \subseteq H \cap B$. Now let $b \in H \cap B$, which implies that $b \in H$ and $b \in B$. Since H is an intra-regular so there exist $x, y \in H$ such that $b \in (x \circ b^2) \circ y$. Now by using paramedial law and left invertive law, we have

$$\begin{aligned} b &\in (x \circ b^2) \circ y = ((e \circ x) \circ (b \circ b)) \circ y = ((b \circ b) \circ (x \circ e)) \circ y \\ &= (((x \circ e) \circ b) \circ b) \circ y \subseteq (H \circ B) \circ H. \end{aligned}$$

Which shows that $(H \circ B) \circ H = H \cap B$. □

The converse is not true in general. For this, let us consider an LA-semihypergroup $H = \{e, x, y, z, w\}$ with the binary hyperoperation defined below:

$\circ$	e	x	y	z	w
e	e	x	y	z	w
x	y	z	z	$\{z, w\}$	w
y	x	z	z	$\{z, w\}$	w
z	z	$\{z, w\}$	$\{z, w\}$	$\{z, w\}$	w
w	w	w	w	w	w

Clearly H is an LA-semihypergroup because the elements of H satisfies the left invertive law. It is easy to see that $\{z, w\}$ is an interior hyperideal of H with pure left identity e such that $(H \circ \{z, w\}) \circ H = \{z, w\} \cap H$ but H is not an intra-regular because $x \in H$ is not an intra-regular.

Corollary 5. *If H is an intra-regular LA-semihypergroup with pure left identity, then $(H \circ B) \circ H = B$, where B is an interior hyperideal of H.*

Let H be an LA-semihypergroup, then $\emptyset \neq A \subseteq H$ is called semiprime if $a^2 \subseteq A$ implies $a \in A$.

Theorem 6. *An LA-semihypergroup H with pure left identity is intra-regular if $L \cup R = L \circ R$, where L and R are the left and right hyperideals of H respectively such that R is semiprime.*

Proof. Let H be an LA-semihypergroup with pure left identity, then clearly $H \circ a$ and $a^2 \circ H$ are the left and right hyperideals of H such that $a \in H \circ a$ and $a^2 \subseteq a^2 \circ H$, because by using paramedial law, we have

$$a^2 \circ H = (a \circ a) \circ (H \circ H) = (H \circ H) \circ (a \circ a) = H \circ a^2.$$

Therefore by given assumption, $a \in a^2 \circ H$. Now by using left invertive law, medial law, paramedial law and Lemma 1, we have

$$\begin{aligned} a &\in H \circ a \cup a^2 \circ H = (H \circ a) \circ (a^2 \circ H) \\ &= (H \circ a) \circ ((a \circ a) \circ H) = (H \circ a) \circ ((H \circ a) \circ (e \circ a)) \\ &\subseteq (H \circ a) \circ ((H \circ a) \circ (H \circ a)) = (H \circ a) \circ ((H \circ H) \circ (a \circ a)) \\ &\subseteq (H \circ a) \circ ((H \circ H) \circ (H \circ a)) = (H \circ a) \circ ((a \circ H) \circ (H \circ H)) \\ &= (H \circ a) \circ ((a \circ H) \circ H) = (a \circ H) \circ ((H \circ a) \circ H) \\ &= (a \circ (H \circ a)) \circ (H \circ H) = (a \circ (H \circ a)) \circ H \\ &= (H \circ (a \circ a)) \circ H = (H \circ a^2) \circ H. \end{aligned}$$

Which shows that H is intra-regular. □

Lemma 4. *If H is an intra-regular LA-semihypergroup, then $H = H^2$.*

Proof. The proof straightforward. □

Theorem 7. *For a left invertible LA-semihypergroup H with pure left identity and $e = a^{'} \circ a$, the following conditions are equivalent.*

(i) *H is intra-regular.*

(ii) *$R \cap L = R \circ L$, where R and L are any right and left hyperideals of H respectively.*

Proof. $(i) \Longrightarrow (ii)$: Assume that H is an intra-regular LA-semihypergroup with pure left identity and let $a \in H$, then there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$. Let R and L be any right and left hyperideals of H respectively, then obviously $R \circ L \subseteq R \cap L$. Now let $a \in R \cap L$ implies that $a \in R$ and $a \in L$. Now by using medial law, left invertive law and Lemma 1, we have

$$\begin{aligned} a &\in (x \circ a^2) \circ y \in (H \circ a^2) \circ H = (H \circ (a \circ a)) \circ H \\ &= (a \circ (H \circ a)) \circ H = (a \circ (H \circ a)) \circ (H \circ H) \\ &= (a \circ H) \circ ((H \circ a) \circ H) = (H \circ a) \circ ((a \circ H) \circ H) \\ &= (H \circ a) \circ ((H \circ H) \circ a) = (H \circ a) \circ (H \circ a) \\ &\subseteq (H \circ R) \circ (H \circ L) = ((H \circ H) \circ R) \circ (H \circ L) \\ &= ((R \circ H) \circ H) \circ (H \circ L) \subseteq R \circ L. \end{aligned}$$

This shows that $R \cap L = R \circ L$.

$(ii) \Longrightarrow (i)$: Let H be a left invertible LA-semihypergroup with pure left identity, then for $a \in H$ there exists $a^{'} \in H$ such that $e = a^{'} \circ a$. Since $a^2 \circ H$ is a right hyperideal and also a left hyperideal of H such that $a^2 \subseteq a^2 \circ H$, therefore by using given assumption, medial law, left invertive law and Lemma 1, we have

$$\begin{aligned} a^2 &\subseteq a^2 \circ H \cap a^2 \circ H = (a^2 \circ H) \circ (a^2 \circ H) = a^2 \circ ((a^2 \circ H) \circ H) \\ &= a^2 \circ ((H \circ H) \circ a^2) = (a \circ a) \circ (H \circ a^2) = ((H \circ a^2) \circ a) \circ a. \end{aligned}$$

Thus we get, $a^2 \subseteq ((x \circ a^2) \circ a) \circ a$ for some $x \in H$.

Now by using left invertive law, we have

$$\begin{aligned} (a \circ a) \circ a^{'} &= (((x \circ a^2) \circ a) \circ a) \circ a^{'} \\ (a^{'} \circ a) \circ a &= (a^{'} \circ a) \circ (((x \circ a^2) \circ a) \\ a &= (x \circ a^2) \circ a. \end{aligned}$$

This shows that H is intra-regular. □

Lemma 5. *Every two-sided hyperideal of an intra-regular LA-semihypergroup H with left identity is idempotent.*

Proof. The proof is straightforward. □

Theorem 8. *In an LA-semihypergroup H with left identity, the following conditions are equivalent.*

(i) *H is intra-regular.*

(ii) *$A = (H \circ A)^2$, where A is any left hyperideal of H.*

Proof. $(i) \Longrightarrow (ii)$: Let A be a left hyperideal of an intra-regular LA-semihypergroup H with left identity, then $H \circ A \subseteq A$ and by Lemma 5, $(H \circ A)^2 = H \circ A \subseteq A$. Now $A = A \circ A \subseteq H \circ A = (H \circ A)^2$, which implies that $A = (H \circ A)^2$.

$(ii) \Longrightarrow (i)$: Let A be a left hyperideal of H, then $A = (H \circ A)^2 \subseteq A^2$, which implies that A is idempotent, hence H is intra-regular. $\square$

Theorem 9. *In an intra-regular LA-semihypergroup H with pure left identity, the following conditions are equivalent.*
(i) *A is a bi-(generalized bi-) hyperideal of H.*
(ii) *$(A \circ H) \circ A = A$ and $A^2 = A$.*

Proof. $(i) \Longrightarrow (ii)$: Let A be a bi-hyperideal of an intra-regular LA-semihypergroup H with pure left identity, then $(A \circ H) \circ A \subseteq A$. Let $a \in A$, then since H is intra-regular so there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$. Now by using medial, left invertive law and Lemma 1, we have

$$\begin{aligned}
a &\in (x \circ a^2) \circ y = (x \circ (a \circ a)) \circ y = (a \circ (x \circ a)) \circ y = (y \circ (x \circ a)) \circ a \\
&= (y \circ (x \circ ((x \circ a^2) \circ y))) \circ a = (y \circ ((x \circ a^2) \circ (x \circ y))) \circ a \\
&= ((x \circ a^2) \circ (y \circ (x \circ y))) \circ a = ((x \circ (a \circ a)) \circ (y \circ (x \circ y))) \circ a \\
&= ((a \circ (x \circ a)) \circ (y \circ (x \circ y))) \circ a = ((a \circ y) \circ ((x \circ a) \circ (x \circ y))) \circ a \\
&= ((x \circ a) \circ ((a \circ y) \circ (x \circ y))) \circ a = ((x \circ a) \circ ((a \circ x) \circ y^2)) \circ a \\
&= ((y^2 \circ (a \circ x)) \circ (a \circ x)) \circ a = (a \circ ((y^2 \circ (a \circ x)) \circ x)) \circ a \subseteq (A \circ H) \circ A.
\end{aligned}$$

Thus $(A \circ H) \circ A = A$ holds. Now by using left invertive law, paramedial law, medial law and and Lemma 1, we have

$$\begin{aligned}
a &\in (x \circ a^2 \circ) \circ y = (x \circ (a \circ a)) \circ y = (a \circ (x \circ a)) \circ y = (y \circ (x \circ a)) \circ a \\
&= (y \circ (x \circ ((x \circ a^2) \circ y))) \circ a = (y \circ ((x \circ a^2) \circ (x \circ y))) \circ a \\
&= ((x \circ a^2) \circ (y \circ (x \circ y))) \circ a = ((x \circ (a \circ a)) \circ (y \circ (x \circ y))) \circ a \\
&= ((a \circ (x \circ a)) \circ (y \circ (x \circ y))) \circ a = (((y \circ (x \circ y)) \circ (x \circ a)) \circ a) \circ a \\
&= (((a \circ x) \circ ((x \circ y) \circ y)) \circ a) \circ a = (((a \circ x) \circ (y^2 \circ x)) \circ a) \circ a \\
&= (((a \circ y^2) \circ (x \circ x)) \circ a) \circ a = (((a \circ y^2 \circ) \circ x^2) \circ a) \circ a \\
&= (((x^2 \circ y^2) \circ a) \circ a) \circ a = (((x^2 \circ y^2) \circ ((x \circ (a \circ a)) \circ y)) \circ a) \circ a \\
&= (((x^2 \circ y^2) \circ ((a \circ (x \circ a)) \circ y)) \circ a) \circ a = (((x^2 \circ (a \circ (x \circ a))) \circ (y^2 \circ y)) \circ a) \circ a \\
&= (((a \circ (x^2 \circ (x \circ a))) \circ y^3) \circ a) \circ a = (((a \circ ((x \circ x) \circ (x \circ a))) \circ y^3) \circ a) \circ a \\
&= (((a \circ ((a \circ x) \circ (x \circ x))) \circ y^3) \circ a) \circ a = ((((a \circ x) \circ (a \circ x^2)) \circ y^3) \circ a) \circ a \\
&= ((((a \circ a) \circ (x \circ x^2)) \circ y^3) \circ a) \circ a = (((y^3 \circ x^3) \circ (a \circ a)) \circ a) \circ a \\
&= ((a \circ ((y^3 \circ x^3) \circ a)) \circ a) \circ a \subseteq ((A \circ H) \circ A) \circ A \subseteq A \circ A = A^2.
\end{aligned}$$

Hence $A = A^2$ holds.
$(ii) \Longrightarrow (i)$ is obvious. $\square$

Theorem 10. *In an intra-regular LA-semihypergroup H with pure left identity, the following conditions are equivalent.*
(i) *A is a quasi hyperideal of H.*
(ii) *$H \circ Q \cap Q \circ H = Q$.*

Proof. $(i) \Longrightarrow (ii)$: Let Q be a quasi hyperideal of an intra-regular LA-semihypergroup H with pure left identity, then $H \circ Q \cap Q \circ H \subseteq Q$. Let $q \in Q$, then since H is intra-regular so there exist $x, y \in H$ such that $q \in (x \circ q^2) \circ y$. Let $p \circ q \subseteq H \circ Q$,

then by using medial law, paramedial law and Lemma 1, we have

$$\begin{aligned} p \circ q &\subseteq p \circ ((x \circ q^2) \circ y) = (x \circ q^2) \circ (p \circ y) \\ &= (x \circ (q \circ q)) \circ (p \circ y) = (q \circ (x \circ q)) \circ (p \circ y) \\ &= (q \circ p) \circ ((x \circ q) \circ y) = (x \circ q) \circ ((q \circ p) \circ y) \\ &= (y \circ (q \circ p)) \circ (q \circ x) = q \circ ((y \circ (q \circ p)) \circ x) \subseteq Q \circ H. \end{aligned}$$

Now let $q \circ y \in Q \circ H$, then by using medial law, paramedial law and Lemma 1, we have

$$\begin{aligned} q \circ p &\subseteq ((x \circ q^2) \circ y) \circ p = (p \circ y) \circ (x \circ q^2) = (p \circ y) \circ (x \circ (q \circ q)) \\ &= x \circ ((p \circ y) \circ (q \circ q)) = x \circ ((q \circ q) \circ (y \circ p)) \\ &= (q \circ q) \circ (x \circ (y \circ p)) = ((x \circ (y \circ p)) \circ q) \circ q \subseteq H \circ Q. \end{aligned}$$

Hence $Q \circ H = H \circ Q$. As by using left invertive law, medial law and Lemma 1, we have

$$\begin{aligned} q &\in (x \circ q^2) \circ y = (x \circ (q \circ q)) \circ y = (q \circ (x \circ q)) \circ y \\ &= (y \circ (x \circ q)) \circ q \subseteq H \circ Q. \end{aligned}$$

Thus $q \in H \circ Q \cap Q \circ H$ implies that $H \circ Q \cap Q \circ H = Q$.

$(ii) \Longrightarrow (i)$ is obvious. □

Theorem 11. *In an intra-regular LA-semihypergroup H with pure left identity, the following conditions are equivalent.*

(i) *A is an interior hyperideal of H.*

(ii) $(H \circ A) \circ H = A$.

Proof. $(i) \Longrightarrow (ii)$: Let A be an interior hyperideal of an intra-regular LA-semihypergroup H with pure left identity, then $(H \circ A) \circ H \subseteq A$. Let $a \in A$, then since H is intra-regular so there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$. Now by using left invertive law, medial law, paramedial law and Lemma 1, we have

$$\begin{aligned} a &\in (x \circ a^2) \circ y = (x \circ (a \circ a)) \circ y = (a \circ (x \circ a)) \circ y \\ &= (y \circ (x \circ a)) \circ a = (y \circ (x \circ a)) \circ ((x \circ a^2) \circ y) \\ &= (((x \circ a^2) \circ y) \circ (x \circ a)) \circ y = ((a \circ x) \circ (y \circ (x \circ a^2))) \circ y \\ &= (((y \circ (x \circ a^2)) \circ x) \circ a) \circ y \subseteq (H \circ A) \circ H. \end{aligned}$$

Thus $(H \circ A) \circ H = A$.

$(ii) \Longrightarrow (i)$ is obvious. □

Theorem 12. *In an intra-regular LA-semihypergroup H with pure left identity, the following conditions are equivalent.*

(i) *A is a $(1, 2)$-hyperideal of H.*

(ii) $(A \circ H) \circ A^2 = A$ *and* $A^2 = A$.

Proof. $(i) \Longrightarrow (ii)$: Let A be a $(1, 2)$-hyperideal of an intra-regular LA-semihypergroup H with pure left identity, then $(A \circ H) \circ A^2 \subseteq A$ and $A^2 \subseteq A$. Let $a \in A$, then since H is intra-regular so there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$. Now by

using left invertive law, medial law, paramedial law and Lemma 1, we have

$$\begin{aligned}
a &\in (x \circ a^2) \circ y = (x \circ (a \circ a)) \circ y = (a \circ (x \circ a)) \circ y \\
&= (y \circ (x \circ a)) \circ a = (y \circ (x \circ ((x \circ a^2) \circ y))) \circ a \\
&= (y \circ ((x \circ a^2) \circ (x \circ y))) \circ a = ((x \circ a^2) \circ (y \circ (x \circ y))) \circ a \\
&= (((x \circ y) \circ y) \circ (a^2 \circ x)) \circ a = ((y^2 \circ x) \circ (a^2 \circ x)) \circ a \\
&= (a^2 \circ ((y^2 \circ x) \circ x)) \circ a = (a^2 \circ (x^2 \circ y^2)) \circ a \\
&= (a \circ (x^2 \circ y^2)) \circ a^2 = (a \circ (x^2 \circ y^2)) \circ (a \circ a) \subseteq (A \circ H) \circ A^2.
\end{aligned}$$

Thus $(A \circ H) \circ A^2 = A$. Now by using left invertive law, medial law, paramedial law and Lemma 1, we have

$$\begin{aligned}
a &\in (x \circ a^2) \circ y = (x \circ (a \circ a)) \circ y = (a \circ (x \circ a)) \circ y = (y \circ (x \circ a))a \\
&= (y \circ (x \circ a)) \circ ((x \circ a^2) \circ y) = (x \circ a^2) \circ ((y \circ (x \circ a)) \circ y) \\
&= (x \circ (a \circ a)) \circ ((y \circ (x \circ a)) \circ y) = (a \circ (x \circ a)) \circ ((y \circ (x \circ a)) \circ y) \\
&= (((y \circ (x \circ a)) \circ y) \circ (x \circ a)) \circ a = ((a \circ x) \circ (y \circ (y \circ (x \circ a)))) \circ a \\
&= ((((x \circ a^2) \circ y) \circ x) \circ (y \circ (y \circ (x \circ a)))) \circ a \\
&= (((x \circ y) \circ (x \circ a^2)) \circ (y \circ (y \circ (x \circ a)))) \circ a \\
&= (((x \circ y) \circ y) \circ ((x \circ a^2) \circ (y \circ (x \circ a)))) \circ a \\
&= ((y^2 \circ x) \circ ((x \circ (a \circ a)) \circ (y \circ (x \circ a)))) \circ a \\
&= ((y^2 \circ x) \circ ((x \circ y) \circ ((a \circ a) \circ (x \circ a)))) \circ a \\
&= ((y^2 \circ x) \circ ((a \circ a) \circ ((x \circ y) \circ (x \circ a)))) \circ a \\
&= ((a \circ a) \circ ((y^2 \circ x) \circ ((x \circ y) \circ (x \circ a)))) \circ a \\
&= ((a \circ a) \circ ((y^2 \circ x) \circ ((x \circ x) \circ (y \circ a)))) \circ a \\
&= ((((x \circ x) \circ (y \circ a)) \circ (y^2 \circ x)) \circ (a \circ a)) \circ a \\
&= ((((a \circ y) \circ (x \circ x)) \circ (y^2 \circ x)) \circ (a \circ a)) \circ a \\
&= ((((x^2 \circ y) \circ a) \circ (y^2 \circ x)) \circ (a \circ a)) \circ a \\
&= (((x \circ y^2) \circ (a \circ (x^2 \circ y))) \circ (a \circ a)) \circ a \\
&= ((a \circ ((x \circ y^2) \circ (x^2 \circ y))) \circ (a \circ a)) \circ a \\
&= ((a \circ (x^3 \circ y^3)) \circ (a \circ a)) \circ a \subseteq ((A \circ H) \circ A^2) \circ A \subseteq A \circ A = A^2.
\end{aligned}$$

Hence $A^2 = A$.

$(ii) \Longrightarrow (i)$ is obvious. □

Lemma 6. *Every non empty subset A of an intra-regular LA-semihypergroup H with pure left identity is a left hyperideal of H if and only if it is a right hyperideal of H.*

Proof. The proof is straightforward. □

Theorem 13. *In an intra-regular LA-semihypergroup H with pure left identity, the following conditions are equivalent.*

(i) *A is a $(1, 2)$-hyperideal of H.*

(ii) *A is a two-sided hyperideal of H.*

Proof. $(i) \Longrightarrow (ii)$: Assume that H is intra-regular LA-semihypergroup with pure left identity and let A be a $(1, 2)$-hyperideal of H then, $(A \circ H) \circ A^2 \subseteq A$. Let $a \in A$,

then since H is intra-regular so there exist $x, y \in H$ such that $a \in (x \circ a^2) \circ y$. Now by using left invertive law, medial law, paramedial law and Lemma 1, we have

$$\begin{aligned}
H \circ a &\subseteq H \circ ((x \circ a^2) \circ y) = (x \circ a^2) \circ (H \circ y) = (x \circ (a \circ a)) \circ (H \circ y) \\
&= (a \circ (x \circ a)) \circ (H \circ y) = ((H \circ y) \circ (x \circ a)) \circ a \\
&= ((H \circ y) \circ (x \circ a)) \circ ((x \circ a^2) \circ y) = (x \circ a^2) \circ (((H \circ y) \circ (x \circ a)) \circ y) \\
&= (y \circ ((H \circ y) \circ (x \circ a))) \circ (a^2 \circ x) = a^2 \circ ((y \circ ((H \circ y) \circ (x \circ a))) \circ x) \\
&= (a \circ a) \circ ((y \circ ((H \circ y) \circ (x \circ a))) \circ x) = (x \circ (y \circ ((H \circ y) \circ (x \circ a)))) \circ (a \circ a) \\
&= (x \circ (y \circ ((a \circ x) \circ (y \circ H)))) \circ (a \circ a) = (x \circ ((a \circ x) \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= ((a \circ x) \circ (x \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= ((((x \circ a^2) \circ y) \circ x) \circ (x \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= (((x \circ y) \circ (x \circ a^2)) \circ (x \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= (((a^2 \circ x) \circ (y \circ x)) \circ (x \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= ((((y \circ x) \circ x) \circ a^2) \circ (x \circ (y \circ (y \circ H)))) \circ (a \circ a) \\
&= (((y \circ (y \circ H)) \circ x) \circ (a^2 \circ ((y \circ x) \circ x))) \circ (a \circ a) \\
&= (((y \circ (y \circ H)) \circ x) \circ (a^2 \circ (x^2 \circ y))) \circ (a \circ a) \\
&= (a^2 \circ (((y \circ (y \circ H)) \circ x) \circ (x^2 \circ y))) \circ (a \circ a) \\
&= ((a \circ a) \circ (((y \circ (y \circ H)) \circ x)(x^2 \circ y))) \circ (a \circ a) \\
&= (((x^2 \circ y) \circ ((y \circ (y \circ H)) \circ x)) \circ (a \circ a)) \circ (a \circ a) \\
&= (a \circ ((x^2 \circ y) \circ (((y \circ (y \circ H)) \circ x) \circ a))) \circ (a \circ a) \subseteq (A \circ H) \circ A^2 \subseteq A.
\end{aligned}$$

Hence A is a left hyperideal of H and by Lemma 6, A is a two-sided hyperideal of H.

$(ii) \Longrightarrow (i)$: Let A be a two-sided hyperideal of H. Let $y \in (A \circ H) \circ A^2$, then $y \in (a \circ H) \circ b^2$ for some $a, b \in A$. Now by using Lemma 1, we have

$$y \in (a \circ H) \circ b^2 = (a \circ H) \circ (b \circ b) = b \circ ((a \circ H) \circ b) \subseteq A \circ H \subseteq A.$$

Hence $(A \circ H) \circ A^2 \subseteq A$, therefore A is a $(1, 2)$-hyperideal of H. □

Lemma 7. *Every non empty subset A of an intra-regular LA-semihypergroup H with pure left identity is a two-sided hyperideal of H if and only if it is a quasi hyperideal of H.*

Proof. The proof is straightforward. □

Theorem 14. *A two-sided hyperideal of an intra-regular LA-semihypergroup H with pure left identity is minimal if and only if it is the intersection of two minimal two-sided hyperideals of H.*

Proof. Let H be intra-regular LA-semihypergroup and Q be a minimal two-sided hyperideal of H, let $a \in Q$. As $H \circ (H \circ a) \subseteq H \circ a$ and $H \circ (a \circ H) \subseteq a \circ (H \circ H) = a \circ H$, which shows that $H \circ a$ and $a \circ H$ are left hyperideals of H, so by Lemma 6, $H \circ a$ and $a \circ H$ are two-sided hyperideals of H. Now

$$\begin{aligned}
& H \circ (H \circ a \cap a \circ H) \cap (H \circ a \cap a \circ H) \circ H \\
= \;& H \circ (H \circ a) \cap H \circ (a \circ H) \cap (H \circ a) \circ H \cap (a \circ H) \circ H \\
\subseteq \;& (H \circ a \cap a \circ H) \cap (H \circ a) \circ H \cap H \circ a \subseteq H \circ a \cap a \circ H.
\end{aligned}$$

This implies that $H \circ a \cap a \circ H$ is a quasi hyperideal of H, so by using 7, $H \circ a \cap a \circ H$ is a two-sided hyperideal of H. Also since $a \in Q$, we have

$$H \circ a \cap a \circ H \subseteq H \circ Q \cap Q \circ H \subseteq Q \cap Q \subseteq Q.$$

Now since Q is minimal, so $H \circ a \cap a \circ H = Q$, where $H \circ a$ and $a \circ H$ are minimal two-sided hyperideals of H, let I be any two-sided hyperideal of H such that $I \subseteq H \circ a$, then $I \cap a \circ H \subseteq H \circ a \cap a \circ H \subseteq Q$, which implies that $I \cap a \circ H = Q$. Thus $Q \subseteq I$. Therefore, we have

$$H \circ a \subseteq H \circ Q \subseteq H \circ I \subseteq I, \text{ gives } H \circ a = I.$$

Thus $H \circ a$ is a minimal two-sided hyperideal of H. Similarly $a \circ H$ is a minimal two-sided hyperideal of H.

Conversely, let $Q = I \cap J$ be a two-sided hyperideal of H, where I and J are minimal two-sided hyperideals of H, then by using 7, Q is a quasi hyperideal of H, that is $H \circ Q \cap Q \circ H \subseteq Q$. Let $Q^{'}$ be a two-sided hyperideal of H such that $Q^{'} \subseteq Q$, then

$$H \circ Q^{'} \cap Q^{'} \circ H \subseteq H \circ Q \cap Q \circ H \subseteq Q, \text{ also } H \circ Q^{'} \subseteq H \circ I \subseteq I \text{ and } Q^{'} \circ H \subseteq J \circ H \subseteq J.$$

Now

$$\begin{aligned} H \circ (H \circ Q^{'}) &= (H \circ H) \circ (H \circ Q^{'}) = (Q^{'} \circ H) \circ (H \circ H) \\ &= (Q^{'} \circ H) \circ H = (H \circ H) \circ Q^{'} = H \circ Q^{'}, \end{aligned}$$

which implies that $H \circ Q^{'}$ is a left hyperideal and hence a two-sided hyperideal by Lemma 6. Similarly $Q^{'} \circ H$ is a two-sided hyperideal of H.

But since I and J are minimal two-sided hyperideals of H, therefore $H \circ Q^{'} = I$ and $Q^{'} \circ H = J$. But $Q = I \cap J$, which implies that, $Q = H \circ Q^{'} \cap Q^{'} \circ H \subseteq Q^{'}$. This give us $Q = Q^{'}$ and hence Q is minimal. □

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